Precalculus

11-03 Cross Products

Cross Product

- $\hat{\imath}$ is _______________ vector in x, $\hat{\jmath}$ is unit vector in y, and $\hat{k}$ is unit vector in z
- $\vec{u} = u_1\hat{\imath} + u_2\hat{\jmath} + u_3\hat{k}$ and $\vec{v} = v_1\hat{\imath} + v_2\hat{\jmath} + v_3\hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

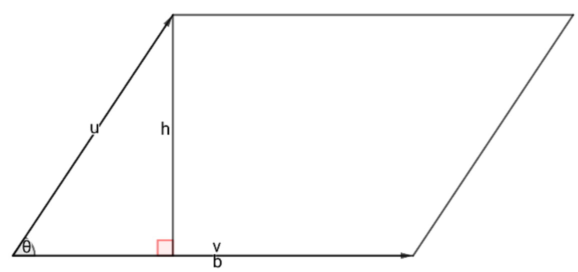
If $\vec{u} = \langle -2,\ 3,\ -3\rangle$ and $\vec{v} = \langle 1,\ -2,\ 1\rangle$, find $\vec{u} \times \vec{v}$

Properties of Cross Products

- $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$
- $\vec{u} \times (\vec{v} + \vec{w}) = (\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w})$
- $c(\vec{u} \times \vec{v}) = c\vec{u} \times \vec{v} = \vec{u} \times c\vec{v}$
- $\vec{u} \times \vec{u} = 0$
- If $\vec{u} \times \vec{v} = 0$, then $\vec{u}$ and $\vec{v}$ are parallel
- $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \cdot \vec{w}$
- $\vec{u} \times \vec{v}$ is orthogonal to $\vec{u}$ and $\vec{v}$
- $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\| \sin\theta$

Area of a Parallelogram

$\|\vec{u} \times \vec{v}\|$ where $\vec{u}$ and $\vec{v}$ represent adjacent sides



Triple Scalar Product (shortcut)

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Volume of Parallelepiped

$V = |\vec{u} \cdot (\vec{v} \times \vec{w})|$ **where $\vec{u}$, $\vec{v}$, and $\vec{w}$ represent adjacent edges**